Online Appendix

Imperfect competition with heterogeneous firms, welfare and misallocation

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A Data

A.1 Merging of the datasets FICUS and FARE

For the analysis we merge the two fiscal firm-level data sets FICUS and FARE, covering the periods from 1994 to 2007, and 2008 to 2019, respectively. Both in FICUS and FARE firms are classified by a 4-digit sector nomenclature "NAF" (nomenclature d'activité française). However, from 2008 onward, the FARE sectoral nomenclature changed: new sectors appeared (some FICUS sectors were split), some FICUS sectors disappeared (were merged into a FARE sector). The FICUS nomenclature was organized according to "NAF 1", while the FARE nomenclature is organized according to "NAF 2". In this study we construct a single data set, 1994 - 2019, by extending the sector nomenclature NAF 2 throughout the whole period. That is, we assign the current 4-digit sector nomenclature NAF 2 retrospectively to all firms observed in FICUS. For firms that are observed both in FICUS and FARE or only in FARE their 4-digit sector according to NAF 2 is known. However, for firms that have exited the market before 2008 we do not know to which NAF 2 4-digit sector they would have belonged to if they had continued their activity. To also classify these firms by the NAF 2 4-digit nomenclature we use the following methodology. We first only look at firms that are observed in both data sets FICUS and FARE. From these observations we build a transition matrix where each row represents a 4-digit sector according to NAF 1 and each column represents a 4-digit sector according to NAF 2. Each cell of the transition matrix contains the number of firms transiting from a specific 4-digit sector in FICUS (NAF 1) to the new 4-digit sector in FARE (NAF 2). Table A1 shows an exemplifying transition matrix, where we chose the NAF 1 4-digit sectors 201A - 205C, i.e. the manufacture of wood and products of wood. For instance it can be seen that there are 2060 firms observed that were classified in FICUS in 201A (first row) and in FARE in the sector 1610 (third column), while there are only 46 observations that were classified in 201A and in FICUS in 0220 (first column). From these observed transition frequencies we then calculate the transition probabilities by simply dividing each element of the matrix by the sum of its corresponding row. That is, the NAF 1 - NAF 2 transition probabilities are calculated by

$$p_{IJ} = \frac{\sum_{n \in I,J}^{N_J} \mathbf{1}_{[n \in I \text{ and } n \in J]}}{\sum_{n \in I}^{N_I} \mathbf{1}_{[n \in I]}},$$

where n is a firm observed in both FICUS and FARE, I and J are specific 4-digit sectors according to NAF 1 and NAF 2, respectively. $\mathbf{1}$ is a dummy variable equal to 1 if the condition in parenthesis is fulfilled. Table A1 shows the observed number of transitions, while Table A2 gives the corresponding transition probabilities. In a second step, firms only observed in FICUS belonging to a specific NAF 1 4-digit sector, are assigned to a NAF 2 (at the 4-digit level), by a random draw with transition probabilities given the row of Table A2.

Table A1: FICUS - FARE: Observed transition frequencies

	NAF 2																		
NAF 1	0220	1392	1610	1621	1622	1623	1624	1629	2223	2512	3101	3109	3319	4329	4332	4391	4399	5610	9524	Total
201A	46	0	2060	5	6	22	35	12	0	0	0	7	0	0	25	24	9	5	0	2256
201B	0	0	498	0	0	0	0	0	0	0	0	0	0	17	4	36	24	0	0	579
202Z	0	0	0	108	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	112
203Z	0	7	33	0	15	1880	8	8	41	26	0	41	0	6	1005	386	34	0	0	3490
204Z	0	0	17	0	0	4	857	6	0	0	0	0	35	0	6	0	0	0	0	925
205A	4	16	10	4	0	21	5	1215	0	0	12	317	0	0	87	0	4	10	156	1861
205C	0	0	0	0	0	0	0	86	0	0	0	0	0	0	0	0	0	0	0	86

Table A2: FICUS - FARE: Transitions probabilities

	NAF 2																		
NAF 1	0220	1392	1610	1621	1622	1623	1624	1629	2223	2512	3101	3109	3319	4329	4332	4391	4399	5610	9524	Total
201A	0.02	0.00	0.91	0.00	0.00	0.01	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00	1.00
201B	0.00	0.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.01	0.06	0.04	0.00	0.00	1.00
202Z	0.00	0.00	0.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.00	1.00
203Z	0.00	0.00	0.01	0.00	0.00	0.54	0.00	0.00	0.01	0.01	0.00	0.01	0.00	0.00	0.29	0.11	0.01	0.00	0.00	1.00
204Z	0.00	0.00	0.02	0.00	0.00	0.00	0.93	0.01	0.00	0.00	0.00	0.00	0.04	0.00	0.01	0.00	0.00	0.00	0.00	1.00
205A	0.00	0.01	0.01	0.00	0.00	0.01	0.00	0.65	0.00	0.00	0.01	0.17	0.00	0.00	0.05	0.00	0.00	0.01	0.08	1.00
205C	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00

A firm refers to a legally independent business unit identified by the SIREN-ID. For consistency, we stick to this definition for all the years and firms. The data provider (INSEE) identified large firm groups by performing "profiling". We ignored profiled groups, but used this information for identifying legal units and preparing the firm-level panel.

A.2 Data cleaning

As mentioned in the main text, the industry for food processing (10), the manufacture of tobacco products (12), and the manufacture of coke and refined petroleum products (19) are excluded from the treated sample. Further, we only keep observations reporting values larger than zero in capital stock (tangible assets), number of employees, materials, and production. Table A3 reports summary statistics of a typical four-digit industry if no data cleaning at all was made. The table shows that, compared to the case with data cleaning (see Table 2 in the main text), the average number of firms is more than doubled, and given by 772. This is mainly due to the inclusion in Table A3 of industry 10 and to a smaller extent by keeping firms reporting zero and missing values in the number of employees. However, the table also shows that firms with less than 10 (500 or more) employees account for about 6.7% (53.0%) of total production, which is very close to the figures presented based on the cleaned sample. Hence, our sample generally matches the main characteristics of the French manufacturing sector.

Table A3: Average statistics of a typical four-digit manufacturing industry without data cleaning[a]

Firm size[b]	# of firms	Share of firms	Share of employees	Share of production	Average cost	Profit rate
0	153	26.06	0.02	3.69	106.50	–6.79
1	59	10.05	0.48	0.35	93.62	4.43
2–4	88	14.99	2.00	1.03	95.66	2.76
5–9	74	12.61	3.99	2.07	94.67	3.17
10–19	52	8.86	5.72	3.36	93.74	3.71
20–49	49	8.35	12.39	8.62	92.74	4.01
50–99	16	2.73	8.88	6.55	93.83	3.08
100–199	9	1.53	10.86	8.82	94.40	2.41
200–499	6	1.02	14.88	13.65	94.41	1.95
500+	3	0.51	40.77	51.38	95.87	1.03
NA	78	13.29	0.00	0.48	100.44	–2.42
Total	587	100.00	99.99	100.00	96.37	1.98

[a] All figures represent averages over all four-digit industries and years (1994–2019). Shares are given in %.
[b] Firm sizes are measured by the number of employees. The group NA represents those firms with missing values in the number of employees.

A.3 Further descriptive statistics

Table A4 shows shares of firms, employees, and production wrt each considered 2-digit industry. The table shows that the manufacturing of metal products (25) represents the biggest industry in terms of the average number of firms and average employment, representing about 23% of all firms and 14% of total employment. Instead, the manufacturing for motor vehicles (29) represents the biggest industry in terms of production, accounting for about 15% of total production. See also De Monte (forthcoming) for more descriptive statistics using similar data.

Table A4: Average statistics by 2-digit manufacturing industry[a]

Industry[b]	# of firms	Share of firms	Share of employees	Share of production	Average cost	Profit rate
11	1098	1.90	1.78	4.01	86.27	3.12
13	2290	3.96	2.93	1.95	91.87	2.68
14	2916	5.04	3.19	1.67	93.76	1.36
15	840	1.45	1.42	0.86	78.09	2.13
16	4417	7.64	2.95	2.03	90.13	4.34
17	1153	1.99	3.37	3.31	95.30	2.96
18	6711	11.61	3.65	1.84	102.94	5.82
20	1907	3.30	7.27	12.61	90.28	0.63
21	333	0.58	3.55	4.35	114.14	1.28
22	3491	6.04	8.52	6.09	94.70	3.49
23	3807	6.59	5.46	5.45	91.08	2.86
24	776	1.34	3.78	5.38	89.92	2.96
25	13569	23.47	13.78	9.31	87.78	6.31
26	2186	3.78	6.41	4.53	157.99	0.42
27	1694	2.93	6.00	5.13	91.85	3.04
28	4448	7.69	8.14	6.93	96.96	1.19
29	1471	2.54	9.87	14.99	95.44	0.12
30	528	0.91	5.35	8.14	96.34	0.40
31	4176	7.22	2.58	1.42	87.21	3.92
Total	57811	100.00	100.00	100.00	93.60	4.00

[a] All figures are based on the cleaned dataset and represent averages over the period 1994–2019. Shares are given in %.
[b] 11-beverages, 13-textiles, 14-wearing apparel, 15-leather/related products, 16-wood/products of wood and cork, 17-paper/paper products, 18-printing/reproduction of recorded media, 20-chemicals/chemical products, 21-pharmaceutical products/preparations, 22-rubber/plastic products, 23-other non-metallic mineral products, 24-basic metals, 25-fabricated metal products, 26-computer, electronic, and optical products, 27-electrical equipment, 28-machinery and equipment, 29-motor vehicles/(semi-) trailers, 30-other transport equipment, 31-furniture.

Table A5 reports the distribution of some variables included in z_{nt} to capture unobserved heterogeneity for the estimation of the cost function (see Section 7 and Assumption 9 in the main text). As in the descriptive statistics section, the table reports averages in a typical 4-digit industry, as well as the distribution of firm sizes over the 1994–2019 period. Beside the average number and the average share of firms, the table reports the share of investing firms, the investment-to-labor ratio, and the average firm age as well as the average number of observed periods (denoted by T_n in the main text). Note that firms' investment, i_{nt}, is given by expenditures in intangible assets, reported in the balance sheets, deflated by the corresponding 2-digit investment price index. Unfortunately, firms' investments are not observed for the specific year 2008. We replace the largest part of these missing values by computing $i_{n2008} = K_{n2009} - (1 - \delta_{2008})K_{n2008}$, where K_{nt} represents firms' intangible assets from the balance sheet, deflated by a corresponding 2-digit price index, and δ_t denotes the capital depreciation rate, likewise calculated at the 2-digit level. It can be seen that the share of investing firms is increasing in firm size, where the share of investing firms with only one employee is given by 60%, whereas almost all firms with 500 and more employees report investments in capital (99%). Regarding the investment-to-labor ratio there seems to be two clusters: one with an investment level of about 6000€ (or 0.06) per worker and another cluster with average investment around 10000€. Considering firms' average age and average number of observed periods, it can be seen that, as expected, both variables are increasing in firm size. That is, while the average age (number of observed periods) of firms with only one employee is given by 12.3 years (4.9 periods), the largest size group, firms reporting 500 and more employees, are on average 31.4 years old (and observed on average for 14.1 periods). Firms' age, a_{nt}, is calculated as the difference between the current year and the date of creation of the firm. Note that firms' age does not necessarily correspond to the number of observed periods as especially small firms often show temporal inactivity and/or drop out of the sample because of missing values. Both variables should represent good proxies to capture unobserved heterogeneity.

Table A5: Further average statistics by 4-digit manufacturing industry[a]

Firm size[b]	# of firms	Share of firms	Share of investing firms	Investment-to-labor ratio	Firm age	# of obs. periods
1	42	13.55	60.08	0.16	12.39	4.87
2–4	73	23.55	69.85	0.07	14.21	7.81
5–9	66	21.29	81.89	0.06	17.45	10.58
10–19	49	15.81	90.20	0.06	20.66	12.31
20–49	47	15.16	94.60	0.06	23.69	12.40
50–99	15	4.84	96.64	0.07	26.49	12.93
100–199	9	2.90	97.66	0.08	27.84	13.32
200–499	6	1.94	98.29	0.10	28.46	13.89
500+	3	0.97	98.69	0.12	31.41	14.16
Total	310	100.01	81.00	0.08	18.55	10.00

[a] All figures are based on the cleaned dataset and represent averages over the period 1994–2019. Shares are given in %.
[b] Firm size is measured by the number of employees.

B Estimation of the output demand function

This section explains how we implement two-ways clustering for the estimation of the coefficient variance matrix. The methods is exposed by Cameron and Miller (2015) and Cameron et al. (2011) and applied to the context of GMM. More formally, we assume that

$$\begin{aligned}
&E\left[\eta_{is}\eta_{it}\right] = \sigma_{iist} \text{ for } |s-t| \leq 1,\\
&E\left[\eta_{it}\eta_{jt}\right] = \sigma_{ijtt},\\
&E\left[\eta_{is}\eta_{jt}\right] = \sigma_{ijst} = 0, \text{ for } i = j \text{ and } |s-t| \geq 2 \text{ and for } i \neq j \text{ and } |s-t| \geq 1.
\end{aligned}$$

As there is no possibility of consistently estimating these parameters, we are instead looking to consistently estimate the variance matrix $V[\widehat{\alpha}]$ of dimension $K \times K$. It is convenient to define the set $\mathcal{S}$ of indices of the dependent random terms:

$$\mathcal{S} = \{i, j, s, t : (i = j, |s-t| \leq 1) \vee (i \neq j, s = t)\}.$$

The cardinality of this set is $I(3T-2) + I(I-1)T = 12628$ and increases with I and T. The GMM weighting matrix is estimated in a first step (using IV estimates $\widehat{\eta}_{it}$) by the inverse of

$$\widehat{\mathbf{B}} = \sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{s=1}^{T}\sum_{t=1}^{T} z_{is}z_{jt}^{\top}\widehat{\eta}_{is}\widehat{\eta}_{jt}\mathbf{1}_{[i,j,s,t\in\mathcal{S}]},$$

where the dummy variable $\mathbf{1}_{[i,j,s,t\in\mathcal{S}]} = 1$ if the indices are included in the set $\mathcal{S}$ and 0 otherwise. An alternative (and easier to code) version of matrix $\widehat{\mathbf{B}}$ is:

$$\widehat{\mathbf{B}} = \mathbf{Z}^{\top}(\widehat{\eta}\widehat{\eta}^{\top} \circ \mathbf{S})\mathbf{Z},$$

where the $IT \times IT$ selection matrix $\mathbf{S}$ has an entry (h, j) equal to one if the random terms η_h and η_j are correlated, and zero otherwise. In our case, only about 4.5% of the elements of $\mathbf{S}$ are nonzero. The Hadamard (term by term) multiplication is denoted by $\circ$. One difficulty comes from the fact that $\widehat{\mathbf{B}}$ is not necessarily positive definite. The same applies to our estimated parameters' variance matrix:

$$V[\widehat{\alpha}] = (\mathbf{X}^{\top}\mathbf{Z}\widehat{\mathbf{B}}^{-1}\mathbf{Z}^{\top}\mathbf{X})^{-1},$$

where the matrices $\mathbf{X}$ and $\mathbf{Z}$ are respectively of dimension $(IT \times K)$ and $(IT \times J)$ with the number of instruments not smaller than the number of regressors $L \geq K$. We follow Cameron et al. (2011) and impose positive definiteness on the parameters variance matrix by setting negative eigenvalues to zero in the eigendecomposition. Here, we actually compare different methods for imposing positive definiteness, by either restricting matrix $\mathbf{S}$, $\mathbf{B}$,$\widehat{\eta}\widehat{\eta}^\top \circ \mathbf{S}$ or $V[\widehat{\alpha}]$ to be positive definite; the results were different, but in all cases the diagonal terms of the restricted variance matrix were much lower than the HAC variance matrix.

C Estimation of the cost function

C.1 Nonlinear least squares optimization procedure

As described in the main text, we estimate the structural parameters of the cost funcition using system NLS, with the to functions describing the firm-level Cournot optimal output supply and the total cost function

$$
\begin{aligned}
y_{nt} &= y^s(p_t, w_{nt}, t, z_{nt}) + \varepsilon^y_{nt} \\
&= \frac{p_t - \gamma^{v_1}(z_{nt})v_1(w_{nt},t) - \eta^{v_1}(z_{nt}) - (\phi_0 Y_t + \phi_Y)\, P'(Y_t)}{\gamma^{v_2}(z_{nt})v_2(w_{nt},t) + \eta^{v_2}(z_{nt}) - \phi_s P'(Y_t)} + \varepsilon^y_{nt}. \\
c_{nt} &= \max\{\gamma^u(z_{nt})u(w_{nt},t) + \eta^u(z_{nt}), 0\} + \gamma^{v_1}(z_{nt})v_1(w_{nt},t)y^s_{nt} + \eta^{v_1}(z_{nt})y^s_{nt} \\
&\quad + \frac{1}{2}\gamma^{v_2}(z_{nt})v_2(w_{nt},t)((y^s_{nt})^2 + \sigma^2_y) + \frac{1}{2}\eta^{v_2}(z_{nt})((y^s_{nt})^2 + \sigma^2_y) + \eta^c(z_{nt}) + \varepsilon^c_{nt},.
\end{aligned}
$$

By modeling the observed cost components u, v_1, and v_2 functionally fully flexible and by introducing the functions γ^u, γ^{v_1}, γ^{v_2}, η^u, η^{v_1}, and η^{v_2} to control for unobserved cost efficiency, modeled according to A9 as a linear function of variables contained in z, we finally obtain a nonlinear system with a total of 138 free parameters to be estimated. Hereby the NLS objective function to be minimized encompasses the sum of squared residuals of both equations defined above. The challenge at hand to obtain a good model fit is to prevent from obtaining parameter estimates representing a local optimum of the objective function, which is a likely result given the high dimensional parameter space. Hence, we proceed in the following.

- **Starting values.** To find reasonably good starting values we first perform a grid search evaluating the objective function at 500,000 (joint) random draws for all parameters. Those (random) parameter values yielding the lowest value of the objective function will be used as starting values.
- **Numerical optimization.** We run the numerical optimization using the **R** package optimx and, more specifically, the optimization routines Nelder-Mead (NM) and BFGS (Nash and Varadhan, 2011; Nash et al., 2023). The numerical optimization is started by NM (using the starting values obtained from the grid search) until a large maximal number of iterations is met (100,000 in many cases). The resulting parameter estimates are then used as starting values for the subsequent optimization using the BFGS optimization method. This iterative procedure, i.e. the changing use of the NM and BFGS method, is repeated until there is no further decrease in the objective function. The resulting parameter estimates of the BFGS optimization are then used as starting values to optimize the system again using the Levenberg-Marquardt algorithm, using the **R** routine nls.lm from the minpack.lm package.

Remark. The optimization routine is very time intensive due to the large number of observations, the large number of free parameters, and the changing use of the BFGS and NM algorithm. Parallelization of the estimation of different 2-digit industries helps to reduce the run time.

C.2 Further results: temporal evolution

Figure C1 shows the evolution of the median of the unobserved variable cost efficiency ($\widehat{\gamma}^v$, solid line) and the RTS (dashed line), where both measures refer to the left y-axis, as well as the RTC (dotted line), referring to the right y-axis and corresponds to the median value (over all firms) of the estimates for $d \ln c/dt$. While the RTC is found to vary somewhat below zero (with most negative value in 1999, given by –0.03 and the highest value in 2005, given by –0.00), the effect on total cost is substantial. It should be noted that the reported median values only include the deterministic changes over time. The stochastic changes in $\widehat{\gamma}^v$ correlated with y and t are not included in this estimate of the reported RTC statistic. This stochastic technological change is estimated by $\widehat{\gamma}^v$, which is found to be increasing over the entire period 1994–2019, corresponding to a total increase in variable cost (at the median) of approximately 20%. The median value of the RTS varies little over time, between 1.03 and 1.06. Even though the median RTS is close to constant, there is substantial heterogeneity around this value (see Table 9 in the main text).

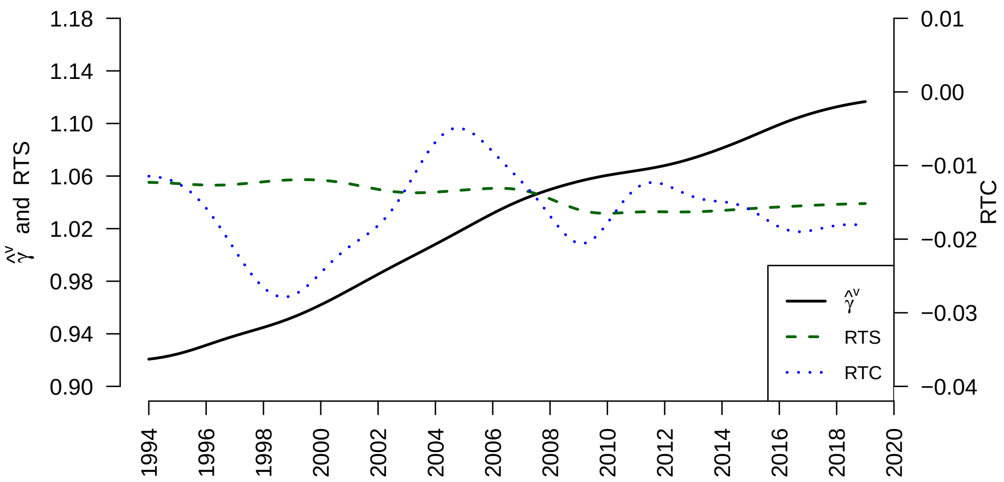

Figure C1: Median evolution of unobserved variable cost efficiency ($\widehat{\gamma}^v$), the rate of Return to Scale (RTS) (both left y-axis), and the Rate of Technological Change (RTC) (right y-axis). The time series are filtered using a kernel-smoother.

D Simulation

Table D1 summarizes the correlation between fitted and observed values obtained over 19 NLS regressions (for each 2-digit industry). The table shows that the NLS estimates provide decent fits, which is necessary for our simulation of output redistribution from less to more productive firms to make sense. In order to reduce computation time and increase prediction accuracy, we only consider firms belonging to the 6 2-digit industries with the best fit between predicted and observed level of production (given by industries 11, 16, 22, 23, 27, and 31, see Table 1 in the main text for a description).

Table D1: Correlation between observed and predicted values

	$cor_N(c_{nt}, \widehat{c}_{nt})$	$cor_N(y_{nt}, \widehat{y}_{nt})$	$cor_N(mr_{nt}, \widehat{\partial c/\partial y})$
Lower quartile	0.88	0.37	0.98
Median	0.92	0.86	0.99
Upper quartile	0.95	0.91	0.99

The correlations are computed for each of the (19) 2-digit industry separately, using industry-specific parameters' estimates. The table reports the quartiles of these 19 correlations.

The purpose of the presented simulation exercise is to compare the economic outcomes of the three different scenarios, namely the Long-Run Cournot Equilibrium (LRCE), the Short-Run Optimal Welfare (SROW), and the Long-run Optimal Welfare (LROW). In particular, as shown in the main text, we aim to compare the (aggregate) production level, price level, welfare, profits, number of firms, and concentration measures. We consider the years 2007–2019 and run the simulation for each year separately. Final results are averages across all years. Further, we only consider the six 2-digit industries for which the best model fit was achieved. In the simulation exercise further explained below we simulate the equilibrium at the level of the 4-digit industries that belong to the selected 2-digit industry. However, in our simulations, we impose the constraint that the production share of each 4-digit sector stays constant at the 2-digit level (for a given year). This constraint avoids the disappearance of specific 4-digit (like "Manufacture of soaps, cleaning products and perfumes") and full replacement by other 4-digit industries (like "Manufacture of pesticides and other agrochemicals").

- **LRCE.** We assume that the data describe a structural economy with Cournot competition. The total observed quantities Y differ form the theoretical values of aggregate Cournot equilibrium Y^C by a sum of random terms (over all firms). We simulate the theoretical Cournot equilibrium Y^C as follows.

  1. Code firms' reaction function based on equation (18), denoted by y^b_{nt}. We impose that the optimal output is included in a window around the observed output level defined by $0 \leq y^b_{nt} \leq 5 \times y_{nt}$. We compute the profit corresponding to the three potential output choices, i.e. the lower bound $\pi(0)$, the inner solution $\pi(y^b_{nt})$, and the upper bound $\pi(5 \times y_{nt})$. This ensures that firms' production level at LRCE, y^C_{nt}, will take reasonable values.
  2. Compute the fixed point according to (8). This determines simultaneously the individual production levels of each firms and the aggregate production level at LRCE for a given 2-digit industry $Y^C_t = \sum_{n=1}^{N} y^C_{nt}$.
  3. Compute the estimated price-level for a given 2-digit industry based on the demand parameter estimates, denoted by P^C_{it} by rearranging equation (29).
  4. Compute profits $\pi^C_{nt} = P^C_{it} y^C_{nt} - c^C_{nt}$, where c^C_{nt} denotes the fitted values of a firm's total cost based on the estimated cost function parameters and the production quantity y^C_{nt} (see equation (33)).
  5. Compute the welfare W^C according to equation (24).

- **SROW.** The SROW represents a scenario in which the central planner removes firms' market power, and obliges firms to set the price equal to their marginal cost. The SROW is simulated by the following the steps:

  1. Compute firms' supply functions at SROW, denoted by y^S_{nt}, by setting firms' marginal cost equal to the market price. Impose that $0 \leq y^S_{nt} \leq 5 \times y_{nt}$.
  2. Compute the fixed point for each 2-digit industry $Y^S_t = \sum_{n=1}^{N} y^S_{nt}$.

3. Compute the estimated price-level for each 2-digit industry based on the demand parameter estimates, denoted by P_{it}^S.
4. Compute profits $\pi_{nt}^S = P_{it}^S y_{nt}^S - c_{nt}^S$.
5. Compute the welfare W^S according to equation (24).

- **LROW.** The LROW represents a scenario in which the central planner is not only able to remove markups but also to replicate the most efficient firm to obtain a better social welfare outcome. The critical task at hand is the choice of the firm to be replicated. For this purpose, we compare three possibilities: replicating the firm at the 75^{th}, at the 90^{th}, or at the 99^{th} percentile of the profit distribution. The LROW is simulated by the following the steps:

  1. Set the percentile α of the profit distribution in turn to $\alpha \in \{0.75, 0.90, 0.99\}$
  2. Compute the initial number of firms $N_{4,i}$ in each 4-digit industry i. To this end, for a given 2-digit industry, we replicate in each 4-digit industry the firm at the α^{th} quantile of the profit distribution a sufficiently large number of times in order to match the observed level of aggregate production in each 4-digit industry. At the starting point, the profit in each 4-digit industry i was positive (abstracting from few exceptions).
  3. Increment the number of firms by a positive unit, chosen to guarantee that the aggregate production share of each 4-digit industry within a given 2-digit industry stays constant throughout the simulation exercise. Repeat this step until the profit is zero in one of the 4 digit industries, this stationary point characterizes the LROW.
  4. Compute the LROW price P_{it}^L, quantity Y_{it}^L, cost and profit c_{nt}^L, π_{nt}^L.
  5. Compute welfare W^L according to equation (24).

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